Araştırma Makalesi
Adıyaman Üniversitesi
Mühendislik Bilimleri Dergisi
19 (2023) 1-14

SEEPAGE ANALYSIS OF A LONG-POLLUTED RIVER IN THE STEADY-STATE USING FINITE DIFFERENCE METHOD

Şeyma Ordu[1]*, Ertuğrul Ordu[2], Reşat Mutlu[3]

[1] Environmental Engineering Namık Kemal University Çorlu, Tekirdağ, Turkey
[2] Civil Engineering Namık Kemal University Çorlu, Tekirdağ, Turkey
[3] Electronics and Telecommunication Engineering Namık Kemal University Çorlu, Tekirdağ, Turkey
Geliş Tarihi/Received Date: 28.07.2022 Kabul Tarihi/Accepted Date: 11.01.2023 DOI: 10.54365/adyumbd.1150091

ABSTRACT
It is very important to analyze the polluted water that streams transfer around them by diffusion. Dirty water leaking from streams like this causes soil and water pollution. The protection of surface and underground water resources is very important given the decreasing water resources. How deep a pollutant has infiltrated can be found by solving the diffusion equation. In this study, the concentration amount of the pollutant passing from a long-polluted stream to the soil was investigated by the finite difference method. Instead of using expensive packet programs, a widespread program Matlab used by engineers and scientists to analyze data, develop algorithms, solve problems analytically and numerically, and create models is preferred in this study. The cross-section of the stream is rectangular and its length is considered to be quite long. Therefore, the problem has been examined in two dimensions. First, the necessary finite difference equations are derived and this problem is analyzed using appropriate boundary conditions. A numerical solution has been obtained for two cases in which the bottom of the stream is permeable and it is not. The mass flow rate of both of the cases was calculated and it is found that the mass flow rate in the first case in which the bottom of the stream is permeable is almost double of the second case. The flow velocities on the boundaries have been found to be less in the case in which the bottom of the stream is impermeable.

Keywords: *Spilled Pollutants Analysis, Finite difference method, River pollution*

UZUN KİRLETİLMİŞ BİR NEHRİN SONLU FARKLAR YÖNTEMİ KULLANILARAK KARARLI HALDE SIZINTI ANALİZİ

ÖZET
Akarsuların etraflarına difüzyon yolu ile aktardıkları kirli suyun analizi oldukça önemlidir. Bu akarsulardan sızan kirli su toprak ve su kirliliğine neden olmaktadır. Yüzey ve yeraltı su kaynaklarının korunması giderek azalan su kaynakları dikkate alındığında çok önemlidir. Bir kirleticinin ne kadar derine sızdığı difüzyon denklemi çözülerek bulunabilir. Bu çalışmada atıksularla kirlenmiş uzun bir dereden toprağa geçen kirleticinin konsantrasyon miktarı sonlu farklar yöntemi ile incelenmiştir. Bu çalışmada pahalı paket programlar kullanmak yerine mühendisler ve bilim insanlarının verileri analiz etmek, algoritma geliştirmek, problemleri analitik ve sayısal olarak çözmek ve model oluşturmak için kullandıkları yaygın bir program olan Matlab tercih edilmiştir. Derenin kesiti diktörtgen alınmış ve boyu oldukça uzun kabul edilmiştir. Bundan dolayı problem iki boyutta incelenmiştir. Önce gerekli sonlu farklar denklemleri türetilmiş ve uygun sınır şartları kullanılarak problem analiz edilmiştir. Akarsuyun tabanının geçirimli olduğu ve olmadığı iki durum için sayısal çözüm elde edilmiştir. Her iki durumun kütlesel debisi hesaplanmış ve akarsuyun tabanının geçirimli olduğu birinci durumdaki kütlesel debinin, ikinci durumun neredeyse iki katı olduğu bulunmuştur. Akarsu tabanının geçirimsiz olması durumunda sınırlardaki akış hızlarının daha az olduğu tespit edilmiştir.

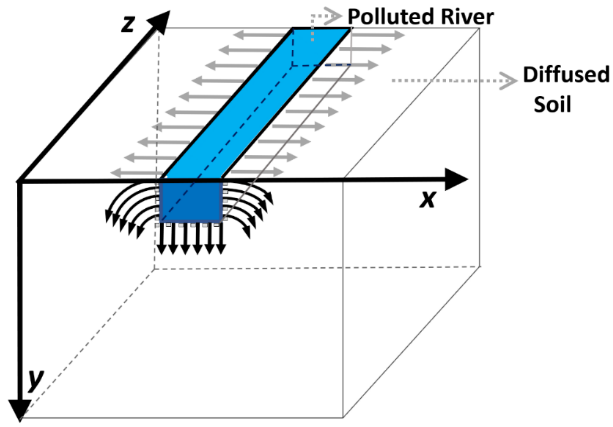
Anahtar Kelimeler: *Dökülen Kirletici Analizi, Sonlu farklar yöntemi, Akarsu kirliliği*

[1]e-posta: sordu@nku.edu.tr ORCID ID: https://orcid.org/0000-0002-5097-2815 (Sorumlu Yazar)
[2]e-posta: eordu@nku.edu.tr ORCID ID: https://orcid.org/0000-0001-8877-5617
[3]e-posta: rmutlu@nku.edu.tr ORCID ID: https://orcid.org/0000-0003-0030-7136

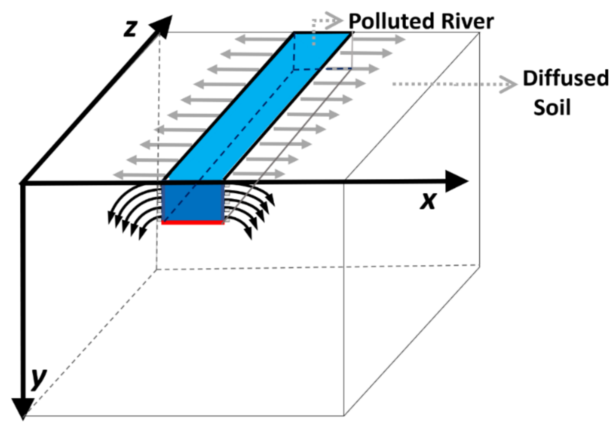
1. Introduction

Water pollution is the mixing of substances that harm the living creatures in the water, threaten human health, prevent economic activities such as fishing, and adversely affect water quality. Due to the increasing industry and population density, pollution is increasing especially in the vicinity of the rivers near the settlements. This pollution is caused by the discharge of domestic and industrial wastewater into rivers without treatment or as a result of insufficient treatment. In addition, wastewater contaminated with pesticides and fertilizers reaches the river beds with the surface flow. As a result, the usability of water resources such as drinking and utility water or agricultural irrigation water decreases considerably [1]. Diffusion is the movement of molecules one by one and randomly, depending on the difference in concentration, from a medium in which they are concentrated to another medium in which they are less dense. Diffusion is expressed by Fick's law [2] . The motion of the molecules is related to the temperature and their kinetic energy is directly proportional to the temperature. As the temperature increases, the rate of movement and diffusion increases [3]. The coefficient used to model the passage of water through the voids in the ground under influence of gravity is called permeability [4] . The permeability coefficient varies according to the porosity and type of soil [5]. The diffusion equation is an equation that describes the diffusion of particles or different physical quantities [6]. The finite difference method is a very useful numerical solution method that provides an approximation for the solution of the diffusion equation. This method is a numerical method based on providing an approximate solution to operations by using the values of a given function at different data points. Finite difference expressions can be easily reached for applications in Cartesian coordinates with defined boundary conditions in two dimensions [6,7]. The finite difference method is the easiest of the numerical method to be applied and therefore it is frequently used in the solution of diffusion equations [8,9]. Polluted waters flowing from streams contaminated with wastewater can penetrate into the surrounding soil [10–12]. Measuring the pollutant concentration within the soil is costly and time-consuming [13]. It is an important value that may define whether the underground water sources are drinkable or not or whether the plants in the region are edible or not. Shackelford presented the equations describing the diffusion process in steady and transient states and the factors affecting diffusion [14]. Experiments which are done for finding pollutant concentration are quite expensive. That's why numerical methods are used instead of performing costly experiments or decreasing the number of experiments required. There are studies in the literature examining the diffusion from water sources to the soil using finite differences because of these reasons. Weeks et al. have used the finite difference method to study pollutant transport in an unsaturated region for the first time [15]. Noye and Tan [16] used an advantageous discretization method with a modified partial differential equation to solve the one-dimensional diffusion equation. Later, they also developed this technique for the two-dimensional diffusion equation [17]. Lardner and Song used the finite difference method to model the two-dimensional and three-dimensional transport due to a point source pollutant [18]. A 9 m long section of a stream with a flow velocity of 0.8 m/s in downstream has been examined and a solution using the finite difference method has been developed in [19]. The work of Kaczmarek et al. showed the effect of diffusion and advection processes on the development of structural damage in clay barriers using a one-dimensional finite difference method [20]. Lastochkin and Favelukis studied a situation in which the diffusion coefficient depends on the concentration using the finite difference method [21]. Kaçur et al. defined a finite difference approach scheme for the solution of pollutant transport problems by diffusion and adsorption in the stability and instability modes [22]. Craig and Rabideau investigated the connection of pollutant transport with water quality in their studies, in which they obtained various solutions with the finite difference method [23]. Sayed et al. developed a distribution model and applied it to the St. Lawrence River [24]. Zhang et al. modeled pollutant transport in saturated soils with the two-dimensional finite difference method [25]. An axisymmetric spilled pollutant problem in steady-state was analyzed with the finite difference method [26]. In this study, the pollutant diffusion from a long-polluted stream with a rectangular cross-section into the surrounding soil has been examined using the finite difference method in the steady state. It is assumed that the soil is homogenous and isotropic. The soil temperature and permeability are taken to be constant for the solution. The diffusion equation for the problem is hard to solve in three dimensions. Since the stream is very long, the effect of the third dimension can be ignored and the problem can be reduced to a two-dimensional one. Even the two-dimensional diffusion equation is hard to solve analytically and, to the best of our knowledge, for such boundary conditions given in this study, it does not have any analytical solutions. Plaxis or Geoslope programs that use the finite element method can be used for the analysis of the problems but they are expensive. Matlab™ is a scientific calculation program used to solve differential equations and matrix systems [27]. That's why 2016a version of Matlab is used in the finite difference solution of the pollutant problem due to its cheapness and availability. The problem has been analyzed using the finite difference method in a Cartesian coordinate system in two dimensions. The distribution of the pollution concentration is normalized and given in two dimensions. The velocity of the pollutant within the surrounding soil is also calculated numerically by post-processing of the solved pollutant concentration using Darcy's law in vector form and is also drawn. The needed plots of the pollutant concentration and velocity within the earth are given. The pollutant problem has been solved for the two cases: the bottom of the stream is permeable in the first one and it is not in the second one. The boundary velocities and the mass flow rate per unit length of both cases are compared for both cases.

This paper is arranged as follows. In the second section, the pollutant problem is described and its finite difference solution is derived. In the third section, the problem is solved and the velocity of the pollutant is also calculated numerically. The solutions of the pollutant concentration and velocity field lines and equipotential concentration lines are given. The paper is finished with the conclusion section.



(a)



(b)

Figure 1. The spilled pollutant problem. a) the bottom of the river is permeable and b) the bottom of the stream is not permeable

2. Definition of the Problem and Derivation of Its Solution with Finite Difference Method

The polluted water or the pollutant is leaking from the stream into the ground as shown in Figure 1. Two cases of pollutant diffusion shown in Figure 1 are considered. In the first case, the bottom of the river is permeable as shown in Figure 1.a. In the second case, the bottom of the river is not permeable, i.e., the pollutant cannot leak into the ground from the bottom of the stream as shown in Figure 1.b. The pollutant leaks into the ground through the stream's rectangular cross-section in both cases. The problem is examined in the x-y plane in both cases. The stream is assumed to be flowing on the z-axis. The soil is assumed to be homogenous and isotropic. Therefore, the problem is two-dimensional. The defined problem is examined using Cartesian geometry. The pollutant density is assumed to be zero under a chosen depth, which can be taken as a big number such as a kilometer, and also at a predefined horizontal distance away from the stream. The predefined horizontal distance is also taken to be one kilometer. It is so often the solution of the different physical problems defined with similar differential equations is found to be the same. For example, a solution to a heat conduction problem may be found in an electromagnetic textbook [28,29]. The diffusion equation has the same form as the heat conduction equation. In this study, the solution to the problem has been inspired by the finite-difference solution of the capacitance of the water sensor given in [30].

If the permeability is constant, the diffusion equation in the Cartesian coordinates is given as

$$\frac{\partial u}{\partial t} = \alpha \Delta u = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{1}$$

In steady state, Eq. (1) turns into

$$\frac{\partial u}{\partial t} = 0 \tag{2}$$

Therefore,

$$\Delta u = 0 \tag{3}$$

In Cartesian coordinates, the Laplacian of the concentration of the pollutant is given as

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{4}$$

where the x, y, and z are dimension components of Cartesian coordinates. The concentration of the pollutant $u(x,y,z)$ within the earth may be taken as only dependent on x and y coordinates since the river is assumed to be very long or the problem can be taken to be two-dimensional. Therefore,

$$\frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial^2 u}{\partial z^2} = 0 \tag{5}$$

Thus, Eq. (4) is reduced to

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{6}$$

Using finite difference, second-order partial derivatives of the pollutant concentration can be approximated as

$$\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x+\Delta x, y) - 2u(x, y) + u(x-\Delta x, y)}{\Delta x^2} \tag{7}$$

and

$$\frac{\partial^2 u}{\partial y^2} \cong \frac{u(x, y+\Delta y) - 2u(x, y) + u(x, y-\Delta y)}{\Delta y^2} \tag{8}$$

By submitting Eq. (5) and (6) into Eq. (4):

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$
$$\cong \frac{u(x+\Delta x, y) - 2u(x, y) + u(x-\Delta x, y)}{\Delta x^2} + \frac{u(x, y+\Delta y) - 2u(x, y) + u(x, y-\Delta y)}{\Delta y^2} = 0 \tag{9}$$

The pollutant concentration of point (x, y) can be found as

$$u(x, y) = \frac{1}{\left(1/\Delta x^2 + 1/\Delta y^2\right)} \left(\frac{u(x+\Delta x, y) + u(x-\Delta x, y)}{2\Delta x^2} + \frac{u(x, y+\Delta y) + u(x, y-\Delta y)}{2\Delta y^2} \right) \tag{10}$$

If $\Delta x = \Delta y$,, the pollutant concentration of point (x, y) becomes

$$u(x, y) = \frac{u(x+\Delta x, y) + u(x-\Delta x, y) + u(x, y+\Delta y) + u(x, y-\Delta y)}{4} \tag{11}$$

The pollutant gradient is calculated as

$$gradu = \frac{\partial u}{\partial x}\vec{e}_x + \frac{\partial u}{\partial y}\vec{e}_y + \frac{\partial u}{\partial z}\vec{e}_z \tag{12}$$

The components of the gradient of the pollutant concentration in the Cartesian coordinates are given as

$$-\frac{\partial u}{\partial x}, \quad -\frac{\partial u}{\partial y} \tag{13}$$

The *z* component of the pollutant concentration is taken to be zero since the finite difference method is applied in two dimensions to the problem. Thus, the pollutant concentration does not depend on *z*-coordinate:

$$\frac{\partial u}{\partial z} = 0 \tag{14}$$

Respectively, the *x* and *y* components of the pollutant concentration gradient can numerically be approximated as

$$-\frac{\partial u}{\partial x} \cong -\frac{u(x+\Delta x, y) - u(x, y)}{\Delta x} \tag{15}$$

and

$$-\frac{\partial u}{\partial y} \cong -\frac{u(x, y+\Delta y) - u(x, y)}{\Delta y} \tag{16}$$

The gradient of the concentration of the pollutant is

$$gradu = \frac{\partial u}{\partial x}\vec{e}_x + \frac{\partial u}{\partial y}\vec{e}_y + \frac{\partial u}{\partial z}\vec{e}_z \tag{17}$$

To calculate the velocity of the pollutant, Darcy's law in vector form is to be used:

$$\vec{v} = -kgrad(u) \tag{18}$$

$$\vec{v} = v_x\vec{e}_x + v_y\vec{e}_y + v_z\vec{e}_z \tag{19}$$

where k is the permeability of the earth, is the pollutant velocity vector and v_x, v_y, and v_z are its components in Cartesian coordinates. v_z is equal to zero due to two-dimensional nature of the problem.

Numerically, the velocity vector components can be calculated as

$$v_x = -k\frac{\partial u}{\partial x} \cong k\frac{u(x,y)-u(x+\Delta x,y)}{\Delta x} \tag{20}$$

and

$$v_y = -k\frac{\partial u}{\partial y} \cong -\frac{u(x,y)-u(x,y+\Delta y)}{\Delta y} \tag{21}$$

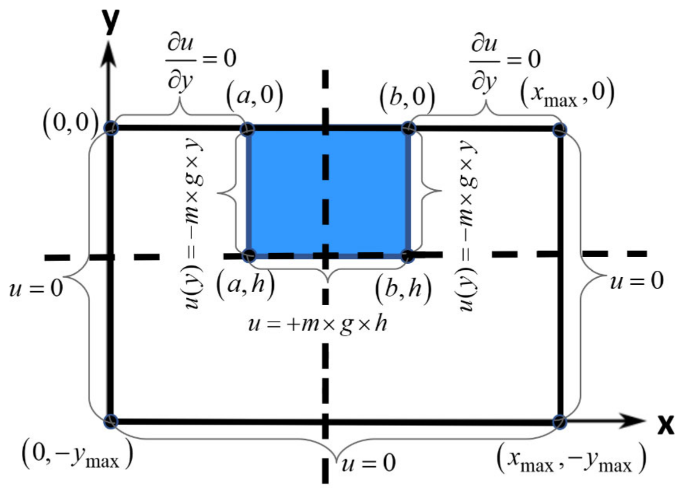
The mass flow rate of the system can be calculated using the surface integral on the outer boundaries of the system:

$$\dot{m} = \iint_S \rho.\vec{v}.d\vec{S} \tag{22}$$

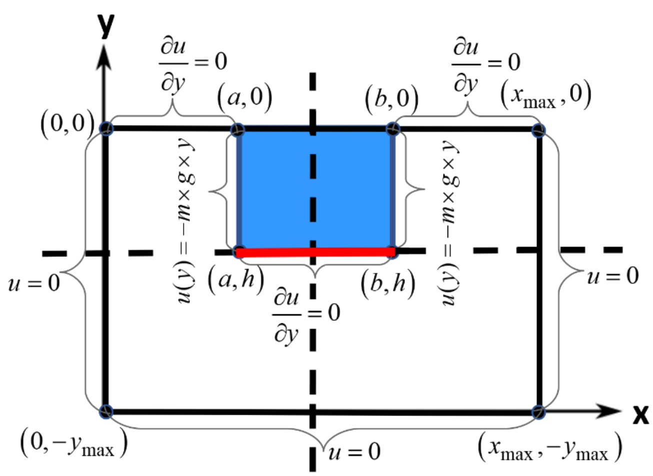
where ρ is the density of the pollutant.

The boundary conditions of both of the cases are shown for two cases in Figure 2. The first case shown in Figure 2.a can be described the follows. The boundary conditions of u or are assumed to be zero at $x=x_{max}$ and at $y=-y_{max}$, which are Dirichlet conditions. At y=0 and for 0<x<a, $\frac{\partial u}{\partial y}$ is zero that is a Neumann condition. At y=0 and for x>b, $\frac{\partial u}{\partial y} = 0$, that is also a Neumann condition. At y=h and for a<x<b, u is constant and equal to mgh that a Dirichlet condition. At x=a and for h>y>0, $u=mgy$ that is a Dirichlet condition. At x=b and for h>y>0, $u=mgy$ that is also a Dirichlet condition.

The second case shown in Figure 2.b can be described the follows. The boundary conditions of u or are assumed to be zero at $x=x_{max}$ and at $y=-y_{max}$, which are Dirichlet conditions. At y=0 and for 0<x<a, $\partial u/\partial y$ is zero that is a Neumann condition. At y=0 and for x>b, $\frac{\partial u}{\partial y} = 0$, that is also a Neumann condition. At y=h and for a<x<b, $\frac{\partial u}{\partial y}$ is constant and equal to zero that is a Neumann condition. At x=a and for h>y>0, $u=mgy$ that is a Dirichlet condition. At x=b and for h>y>0, $u=mgy$ that is also a Dirichlet condition.



(a)



(b)

Figure 2. The definition and the boundary conditions of the problem for the case a) the bottom of the river is permeable and b) the bottom of the stream is not permeable

A flowchart of the solution algorithm is given in Figure 3. The code first produces the concentration matrix. It starts the iteration with Gauss-Seidel Method by scanning the grid points and calculates the point considering the boundary conditions. If the point is not on any boundary Eq. 11 is used to calculate its value. If the point is on a Dirichlet boundary, its calculation is skipped. If it is on a Neumann boundary, the value of the closest point on the parallel line to the Neumann boundary is assigned to its value. The algorithm scans all the points till their relative error values fall under the predefined value of 0.5%. A code is written in Matlab 2016a to simulate the diffusion phenomenon. It is well-known that the finite difference method converges for Laplace and Poisson equations. If an advection term would exist, its convergence should have been verified but the advection term did not exist in our equations in this study.

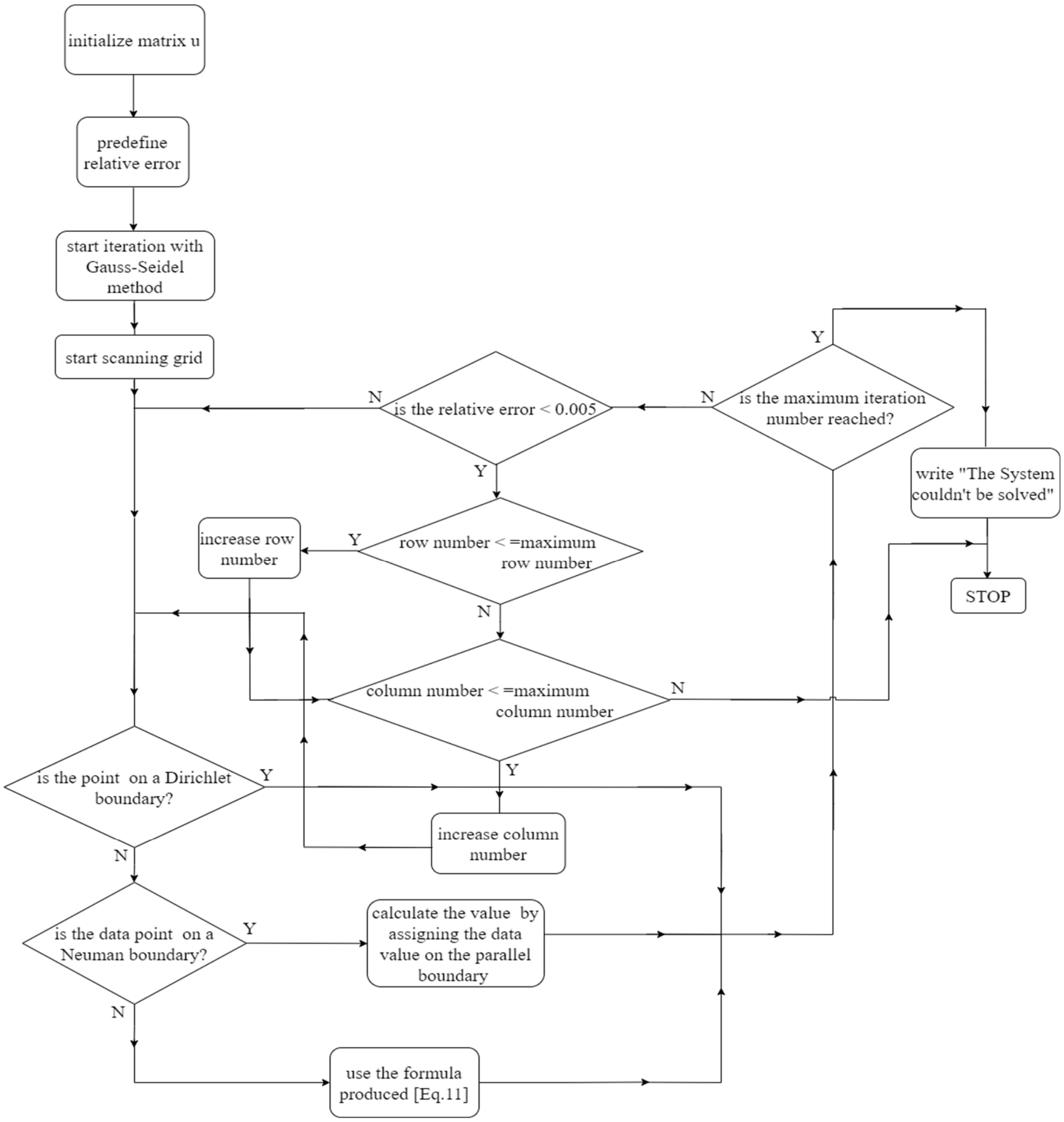


Figure 3. Flowchart of the solution algorithm.

3. Simulation Results and Discussion

Two cases given in Figure 1 are considered in this study. The written code is run for both of the cases. For the first case, the normalized pollutant concentration distribution is shown in Figure 4. The pollutant concentration is linearly dependent on the stream's height on the interface boundaries of the stream and the soil. It is zero at the ground level. The pollutant concentration decreases starting from the interface boundaries of the stream and the soil and falls down considerably at a distance of around half of the stream width as shown in Figure 4. There is a ripple in the pollutant concentration on the ground level near the stream on the Neumann boundaries. The pollutant concentration is maximum on the edges of the stream since the pressure is also maximum on the edges. The velocity field lines for the first case are shown in Figure 5.a. The pollutant velocity is much higher on the edges of the stream and it is maximum at bottom of the stream. On the left and right boundaries, the horizontal pollutant velocity is maximum on the surface (at y=0), starts decreasing with a small slope till around the height of the stream, and then starts falling with a higher slope and reaches zero at $y=-y_{max}$. The vertical pollutant speed on the bottom boundary is almost parabolic and it becomes maximum in the middle of the bottom boundary. The maximum pollutant speed on the bottom boundary is almost triple of the maximum speed on the left and right boundaries. The color map of the pollutant concentration within the soil is shown in Figure 5.b. The pollutant concentration is higher in the regions around and close to the stream as can be seen from the bright colors in the colormap.

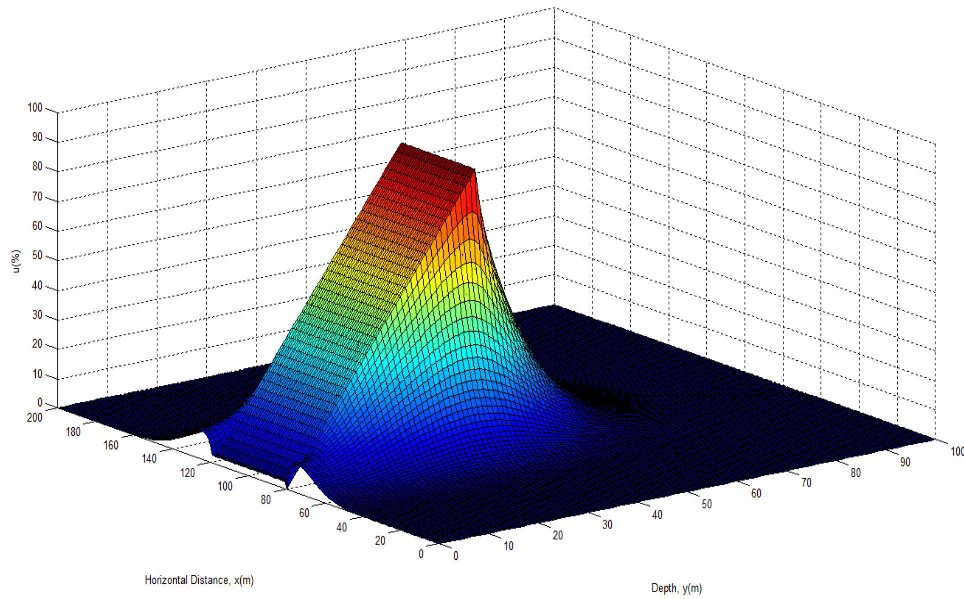
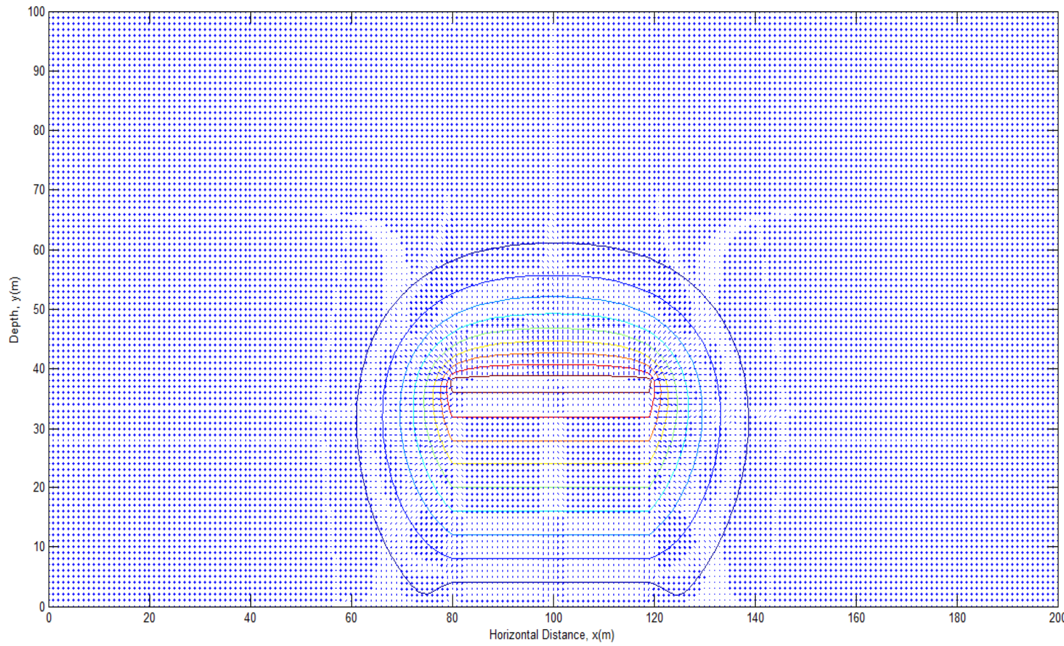
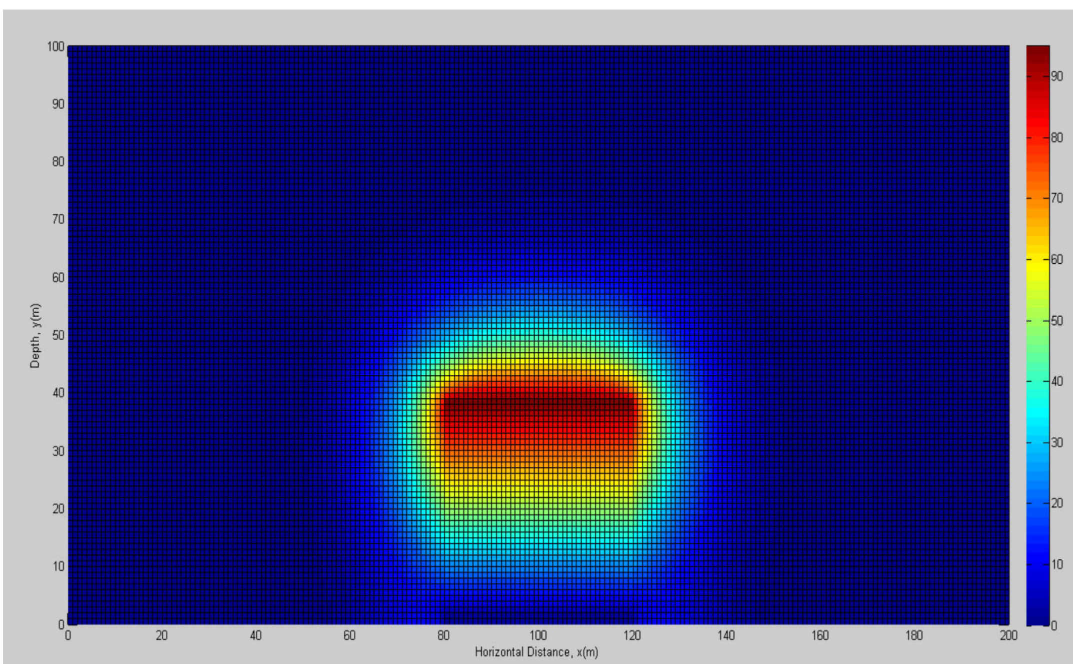


Figure 4. The concentration distribution versus the horizontal distance and height within the soil for the case the bottom of the river is permeable

For the second case, the normalized pollutant concentration distribution is shown in Figure 6. The pollutant concentration is linearly dependent on the stream's height on the interface boundaries of the stream and the soil also in this case except for the bottom of the stream which is impermeable. It is zero at the ground level. The pollutant concentration decreases starting from the left and right interface boundaries of the stream and the soil and falls down considerably at a distance of around half of the stream width as shown in Figure 6. It gets more concentrated as the water depth increases. In this case, there is also a ripple in the pollutant concentration on the ground level near the stream on the Neumann boundaries. The pollutant concentration is also maximum on the edges of the stream in this case since the pressure is also maximum on the edges even though the bottom of the stream is impermeable. The velocity field lines for the second case are shown in Figure 7.a. The pollutant velocity is much higher on the edges of the stream and it is zero on the impermeable bottom of the stream. The pollutant velocity increases as the water depth increases. It is maximum on the edges of the stream but the tangential velocity is zero on the bottom of the stream since it is not permeable. On the left and right boundaries, the horizontal pollutant velocity is maximum on the surface (at y=0), starts decreasing with a small slope till around the height of the stream, and then starts falling with a higher slope and reaches zero at $y=-y_{max}$. The vertical pollutant speed on the bottom boundary is almost parabolic and it becomes maximum in the middle of the bottom boundary. The maximum pollutant speed on the bottom boundary is just 20% higher than the maximum speed on the left and right boundaries as shown in Figure 7.a. The pollutant concentration is higher in the regions around and close to the stream as can be seen from the bright colors in the colormap in Figure 7.b. However, in this case, it is the lowest on the bottom of the stream as seen from the blue color since it is impermeable.



(a)



(b)

Figure 5. a) The equi-concentration surfaces and velocity field lines within the soil and b) Colormap of the concentration for the case the bottom of the river is permeable

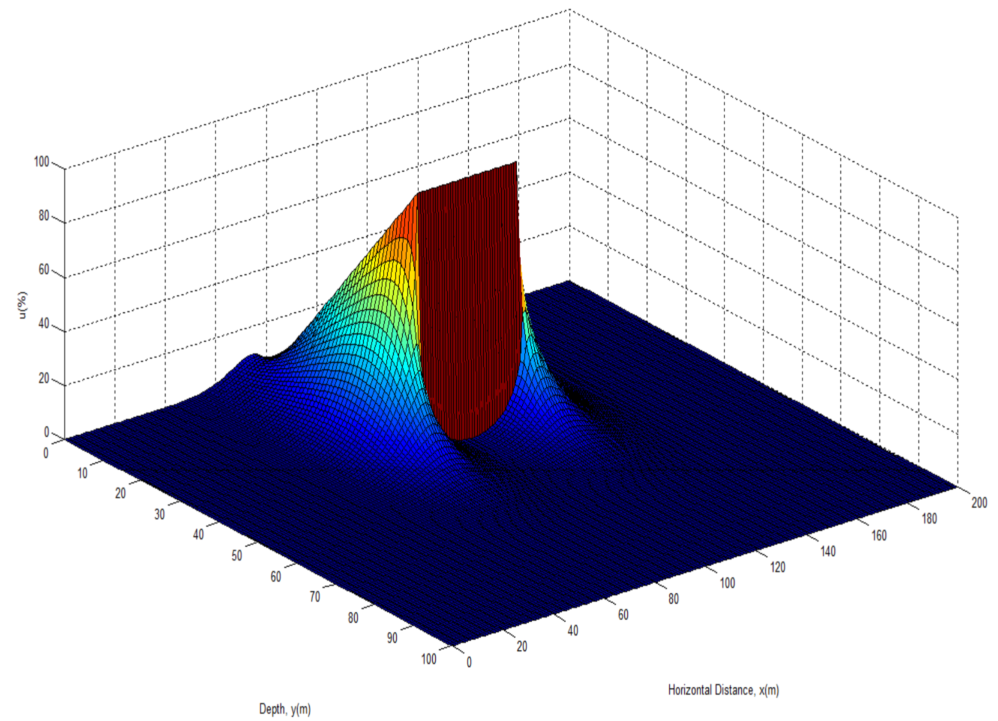
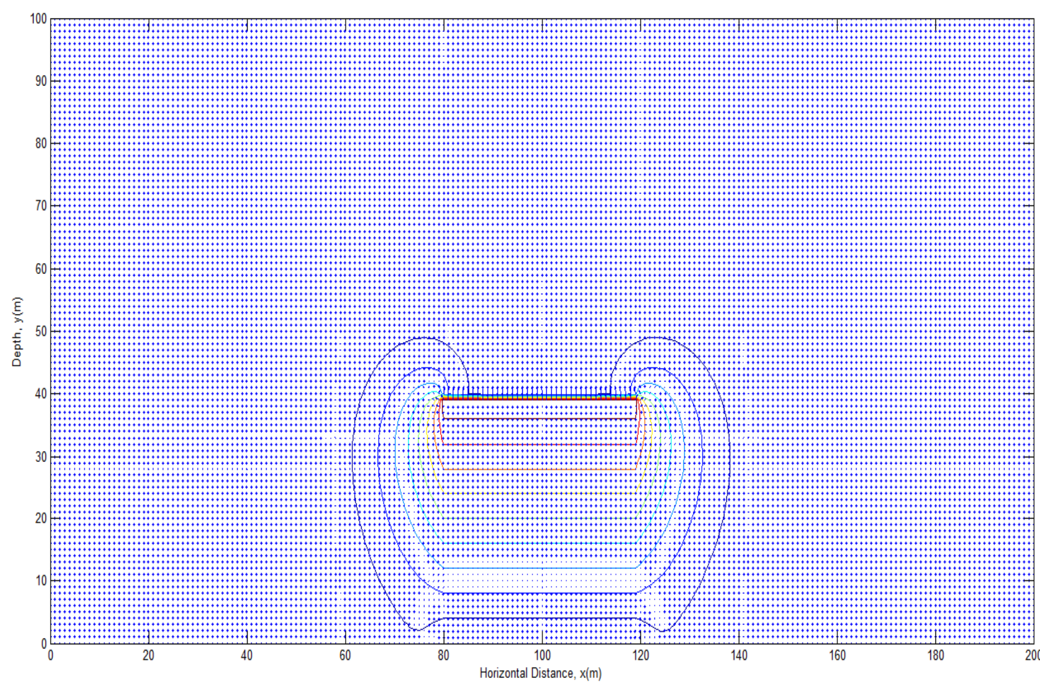
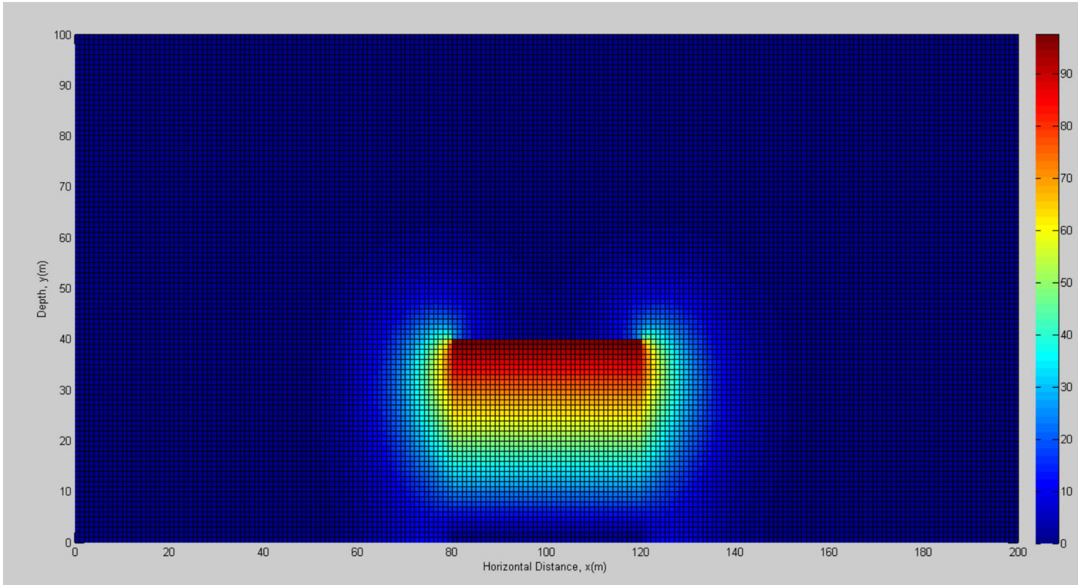


Figure 6. The concentration distribution versus the horizontal distance and height within the soil for the case the bottom of the river is not permeable



(a)



(b)

Figure 7. a) The equi-concentration surfaces and velocity field lines within the soil and b) Colormap of the concentration for the case the bottom of the river is not permeable

The mass flow rate of both of the cases is calculated and it is found that the mass flow rate is almost double of the second case. The maximum speed on the left and right boundaries was almost the same in both cases. However, the average speed on the left and right boundaries in the first case was almost twice of the second case. The maximum speed on the bottom boundary in the first case was almost 50% higher than in the second case. The average speed on the bottom boundary in the first case in the first case was slightly greater than five times in the second case. The results of the comparison indicate that the values are lower if the bottom of the stream is impermeable.

4. Conclusion

Examination of pollutant diffusion within the soil is a very important problem. The Laplace equation in two dimensions in the steady-state is solved using the finite difference method assuming that the soil temperature and permeability are constant, and the soil is homogenous and isotropic. Then, the gradient of the pollutant concentration is calculated numerically and the pollutant speed is calculated using the permeability and its gradient. The mass flow rate has also been calculated by numerical integration of the pollutant concentration velocity on the boundary area. In this paper, instead of using expensive packet programs, using Matlab is suggested for its cheapness and widespread availability to solve the seepage problem. Its numerical and visualization abilities can easily overcome the tasks required in this study. A two-dimensional pollutant diffusion problem from a stream into the soil using finite differences is solved for two cases. The bottom of the stream is permeable in the first case and vice versa. Their comparisons are made. The mass flow rate in the first case is found to be twice the impermeable case. Also, the pollutant velocities on the left, right, and bottom boundaries of the examined problem are found to be less in the second case. This has been expected since the pollutant would leak less into the ground due to the decrease of the leakage area since the bottom of the stream is not permeable.

The soil is actually not a homogenous medium. This method given here can be extended to be used in more sophisticated diffusion problems perhaps with nonhomogeneous soil regions and irregular boundary and interface boundary conditions.

Conflict of Interest Statement

The authors of the article declare that they do not have any personal or financial conflicts of interest with any institution, organization, or person.

References

[1] Davis ML, Cornwell DA. Introduction to environmental engineering. WCB McGraw-Hill; 1998.
[2] Flynn G, Yalkowsky SH, Roseman T. Mass transport phenomena and models: theoretical concepts. Journal of Pharmaceutical Sciences 1974;63:479–510.
[3] Desai CS, Zaman M. Advanced geotechnical engineering: Soil-structure interaction using computer and material models. CRC Press; 2013.
[4] Das BM, Das B. Advanced soil mechanics. vol. 270. Taylor & Francis New York; 2008.
[5] Wanielista M, Kersten R, Eaglin R. Hydrology: Water quantity and quality control. John Wiley and Sons; 1997.
[6] Das BM, Sobhan K. Principles of geotechnical engineering, Cengage Learning. Stamford, Connecticut 2010.
[7] Kiusalaas J. Numerical methods in engineering with Matlab®. Cambridge university press; 2005.
[8] Chatwin P, Allen C. Mathematical models of dispersion in rivers and estuaries. Annual Review of Fluid Mechanics 1985;17:119–49.
[9] Dehghan M. Weighted finite difference techniques for the one-dimensional advection–diffusion equation. Applied Mathematics and Computation 2004;147:307–19.
[10] Ott WR. Environmental statistics and data analysis. Routledge; 2018.
[11] Tychon B, Vander Borght P, De Backer LW. Water and nitrogen transfer study through soils of a small agricultural water catchment. Water Science and Technology 1999;39:69–76.
[12] Johnes PJ. Evaluation and management of the impact of land use change on the nitrogen and phosphorus load delivered to surface waters: the export coefficient modelling approach. Journal of Hydrology 1996;183:323–49.
[13] Woitke P, Wellmitz J, Helm D, Kube P, Lepom P, Litheraty P. Analysis and assessment of heavy metal pollution in suspended solids and sediments of the river Danube. Chemosphere 2003;51:633–42.
[14] Shackelford CD. Diffusion of contaminants through waste containment barriers. Transportation Research Record 1989.
[15] Weeks EP, Earp DE, Thompson GM. Use of atmospheric fluorocarbons F-11 and F-12 to determine the diffusion parameters of the unsaturated zone in the southern high plains of Texas. Water Resources Research 1982;18:1365–78.
[16] Noye B, Tan H. A third-order semi-implicit finite difference method for solving the one-dimensional convection-diffusion equation. International Journal for Numerical Methods in Engineering 1988;26:1615–29.
[17] Noye B, Tan H. Finite difference methods for solving the two-dimensional advection–diffusion equation. International Journal for Numerical Methods in Fluids 1989;9:75–98.
[18] Lardner R, Song Y. An algorithm for three-dimensional convection and diffusion with very different horizontal and vertical length scales. International Journal for Numerical Methods in Engineering 1991;32:1303–19.
[19] Sankaranarayanan S, Shankar N, Cheong H. Three-dimensional finite difference model for transport of conservative pollutants. Ocean Engineering 1998;25:425–42.
[20] Kaczmarek M, Hueckel T, Chawla V, Imperiali P. Transport through a clay barrier with the contaminant concentration dependent permeability. Transport in Porous Media 1997;29:159–78.
[21] Lastochkin D, Favelukis M. Bubble growth in a variable diffusion coefficient liquid. Chemical Engineering Journal 1998;69:21–5.
[22] Kačur J, Malengier B, Remešíková M. Solution of contaminant transport with equilibrium and non-equilibrium adsorption. Computer Methods in Applied Mechanics and Engineering 2005;194:479–89.
[23] Craig JR, Rabideau AJ. Finite difference modeling of contaminant transport using analytic element flow solutions. Advances in Water Resources 2006;29:1075–87.
[24] Sayed M, Serrer M, Mansard E. Oil spill drift and fate model. Oil Spill Response: A Global Perspective, Springer; 2008, 205–20.
[25] Zhang Y, Wang Q, Zhang ST. Numerical simulation of Benzene in soil contaminant transport by finite

difference method. vol. 414, Trans Tech Publ; 2012, 156–60.
[26] Ordu S, Ordu E, Mutlu R. Axisymmetric spilled pollutant analysis in steady-state using finite difference method. Feb-Fresenius Environmental Bulletin 2022:9587-9592.
[27] Gündoğdu Ö, Kopmaz O, Ceviz MA. Mühendislik ve fen uygulamalarıyla Matlab. Nobel Yayıncılık; 2004.
[28] Halliday D, Resnick R, Walker J. Fundamentals of physics. John Wiley & Sons; 2013.
[29] Cengel YA. a Afshin J. Ghajar A. Heat and Mass Transfer, 4th Edition in SI Units, Special Indian Edition, Mcgraw Hill Education, New York, United States, 2011.
[30] Karaoğlan HC, Mutlu R. A Water Level Sensor Design Using Finite Difference Solution and Its Coding in Matlab. European Journal of Engineering and Applied Sciences. 2020; 3(1), 37-45.